Coins tossing

Marina Cazzola

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An interactive version of this document is available here: WIMS : Coins tossing

1 The game

Let's suppose we can toss a fair coin as many times as we want (i.e. a coin that gives heads and tails both with probability equal to 1/2).

The first player chooses a triplet (e.g. *THH*), afterwards the second player chooses another triplet (e.g. *HTH*). The coin is tossed until one of the chosen triplets appears. The player whose triplet appears first wins.

With the choices above, if the sequence of tosses is *HTTTHTTHH*, then the first player wins because the triplet *HTH* did not show up, while the triplet *THH* appears in the last three tosses. Instead, if the sequence of tosses is *HTTHTH*, the second player wins.

Keep in mind that when we start the game, we have no way of knowing how many tosses will be needed to end the game.

1.1 A single game

> **Esercizio interattivo 1**
>
> WIMS : A single game: here you can practice with the game rules.

1.2 You choose first

We will start playing 5 games. You choose a triplet, afterwards the computer chooses its triplet.

> **Esercizio interattivo 2**
>
> WIMS : You choose first (5 games): Choose a triplet.

1.2.1 Some thoughts

Whatever triplet you choose, the computer will always be able to choose one with a higher probability of winning than yours.

Remember this is a random experiment: even if the computer have chosen a triplet with a higher probability than yours, still luck matters. The computer could actually turn out to be unlucky and lose the game.

Have a look at the long run: let's play 20 games.

> **Esercizio interattivo 3**
>
> WIMS : You choose first (20 games): Let's play 20 games.

1.2.2 How does it works?

You can play many times trying to understand the strategy the computer uses to choose its triplets.
Also, you can try to find the triplets that can better challenge the computer's luck (and maybe you can even win).

> **Esercizio interattivo 4**
>
> WIMS : You choose first (20 games): Let's play 20 games and afterwards let's have a look at all the outcomes.

1.3 The computer chooses first

Now the computer will make the first choice. If you have understood which are the "weakest" triplets, in this way you can try to identify the triplet that best responds to the computer's choice.

> **Esercizio interattivo 5**
>
> WIMS : The computer chooses first (5 games): The computer first chooses a triplet and you can respond by choosing yours. Try to find out which triplet can beat the one chosen by the computer.

> **Esercizio interattivo 6**
>
> Have a look at the long run: let's play 20 games.
> WIMS : The computer chooses first: (20 games): let's play 20 games.

1.4 Let's work on the strategy

Now we will try to figure out how to make educated guesses in playing the game. Let's recap a bit of facts about probability.
We will start with the easiest tasks and then we will work on more intriguing problems.

1.4.1 Probabilities

Let's start with the computation on the easier probabilities

> **Esercizio interattivo 7**
>
> WIMS : Probability of a triplet

> **Esercizio interattivo 8**
>
> WIMS : Comparing probabilities

1.4.2 Some special cases

> **Esercizio interattivo 9**
>
> WIMS : Stages in the game

Esercizio interattivo 10

WIMS : Triplet comparisons

1.4.3 Further explicit computations

Esercizio interattivo 11

WIMS : Four tosses